28.84% nitroglycerine without having to handle nitroglycerine directly. It can be seen from Table 1 that the ν-OH obtained for a given nitrocellulose sample was essentially the same as for the corresponding nitroglycerine-nitrocellulose mixture. This then indicates that the nitroglycerine does not hydrogen bond with the hydroxyl groups present in nitrocellulose. This finding supports a similar conclusion drawn in a previous study (Brodman et al., 1974b) concerning hydrogen bonding of nitrocellulose nitrate ester groups to hydroxyl groups.

It should be pointed out that the ν-OH of nitrocellulose has been found to vary from specific sample to sample depending in part on the nitrogen content and molecular weight (Brodman et al., 1974b). This effect can be observed in the present data since each nitrocellulose sample has a different ν-OH. However, essentially the same ν-OH was obtained when nitroglycerine was present.

Implications of these findings on the small arms deterring process will now be discussed. The fact that ethyl acetate hydrogen bonds to unesterified hydroxyl groups in nitrocellulose and that the bond strength for this interaction is the same as that for the di-n-butyl phthalate-nitrocellulose interaction is significant in terms of the deterring process. This indicates that residual solvent in the grain can cause the deterrent to move in deeper by blocking available hydroxyl groups. Nitroglycerine does not hydrogen bond to nitrocellulose and therefore should not affect deterrent depth of penetration by bonding to unesterified hydroxyl groups.

Thus it can be seen that, for a given set of process conditions (time, temperature, and quantity), the transport phenomena can be altered through exploitation of the interactions involved.

LITERATURE CITED

1. Brodman, B. W., M. P. Devine, and M. T. Gurbarg, "Hydrogen Bonding of Deterrents to Unesterified Hydroxyl Groups in Nitrocellulose," *J. Appl. Polym. Sci.*, **18**, 943 (1974a).
2. ———————., "Hydrogen Bonding in Nitrocellulose and its Implications on Deterring of Small Arms Propellant," *J. Macromolecular Sci.-Chem.*, in press (1974b).
3. Maag, R. H., U.S. Pat. 3,037,891, "Smokeless Powder" (1962).
4. Herzog, M. L., U.S. Pat. 2,394,449, "Manufacture of Explosives" (1964).

Manuscript received February 26, 1974; revision received March 26 and accepted April 12, 1974.

Entry Region for Steady Viscous Flow in Coiled Circular Pipes

LARRY R. AUSTIN and J. D. SEADER

Department of Chemical Engineering
The University of Utah, Salt Lake City, Utah 84112

Coiled circular pipes are widely used in fluid flow, heat transfer, mass transfer, and chemical reaction applications. Consequently, a number of experimental and analytical studies concerning curved-tube transport phenomena have been reported as summarized by Austin (1971) and Kalb (1973). The viscous flow region has been of particular interest because of the significant departure of the transport coefficients from the values for straight pipe flow. However, almost all of the reported work has dealt with the case of steady state, fully developed flow.

In some applications, only partial or single coils may be used or the entry region of a multiple coil pipe may be critical; therefore, it is of interest to inquire into the nature of the flow in the entry region and the distance or degrees around the coil required to achieve essentially fully developed flow.

Keulegan and Beij (1937) collected pressure drop data on tubes of approximately 0.95 cm. I.D. for very gradual bends. Data were taken at points along the length of the coil in order to determine when the curved-tube pressure drop ceased to change with distance. They were able to predict an entrance length only under the assumption that it was not a function of the Reynolds number. Under this assumption, they were able to produce an empirical correlation for the entrance length as a function of the curvature ratio only. The purpose of this note is to present the results of a more detailed experimental study of the entry region under steady viscous flow conditions.

EXPERIMENTAL APPARATUS

The experiments were conducted with air flowing through coiled pipes constructed by Ilco Tubebending, Inc. from 4.445-cm. I.D. stainless steel pipe of 0.3175-cm. wall thickness. Pipes of four different curvature ratios (ratio of coil diameter to tube inside diameter) were used, namely 6.94, 9.06, 14.4, and 24.1. Each pipe had at least one complete coil turn. The air flow was first directed through a long straight pipe in order to establish a parabolic velocity profile before entering the coil. Coil pitch (ratio of distance between successive coil turns to

Correspondence concerning this note should be addressed to J. D. Seader. L. R. Austin is with E. I. du Pont de Nemours and Company, Chattanooga, Tennessee.

coil circumference) was not a variable, but was kept approximately constant at a value of 0.065. Experimental data were taken in the form of diametrical axial-velocity profiles measured at various positions around the coil by means of a hot-film anemometry system. These profiles were obtained both along a diameter in the direction of the radius of curvature and along a diameter perpendicular to the radius of curvature by a specially constructed probe-positioning unit. The design of this unit was centered around a commercial circular degree scale. Flexibility of probe position and orientation was provided by four degrees of freedom of movement: 2 in. in and out, 15° from normal to pipe axis, 45° in a plane normal to pipe axis, and 360° rotation. Each measurement port was provided with a securely fastened positioning plate for mounting the probe-positioning unit. The accuracy in moving the probe from one point to the next was within 0.05 mm. The reproducibility after completely disassembling the probe and unit was within 0.12 mm.

Voltage signals from the anemometer were recorded with a digital voltmeter. The four significant figures obtained for the signals were ample to provide very smooth and reproducible results. The voltage signals were converted to velocity values by calibration curves which were developed by measuring known parabolic velocity distributions in the straight-pipe section just upstream of the coil section. The accuracy of the velocity measurement was estimated to be within five percent.

A detailed description of the experimental equipment, and its calibration and operation are given by Austin (1971).

EXPERIMENTAL RESULTS

Experimental data were obtained over a Dean number (Reynolds number divided by the square root of the curvature ratio) range of 198 to 948. A good qualitative description of the flow development can be obtained by considering one set of axial velocity profiles for a Dean number of 530 and a curvature ratio of 14.4 as shown in Figures 1 and 2. Data for other coils and conditions are given by Austin (1971).

Figure 1 shows seven axial velocity profiles each measured across the pipe diameter along the coil radius. The data points for a given profile are plotted from left to right in moving from the inside of the coil to the outside of the coil. The first profile, at the bottom of the figure, was measured just at the entry to the coil. Typically, a parabolic velocity profile was observed. The second profile, taken at 22.5° around the coil turn shows the maximum velocity being crowded towards and very near the outside of the coil. This type of behavior could be expected since the coiling process has now placed the outside wall of the coil directly in line with the central axis of the straight-pipe section. Thus, the inertia of the fluid has carried the maximum centerline velocity in the straight section toward the outer wall of the coil as it starts around the bend. Then by continuity, the velocity at the inside of the coil would tend to diminish, since as the central core moved over near the wall, the fluid with less velocity near the outer wall would be forced around toward the inside of the coil. This marks the beginning of the appearance of the typical secondary velocity found in fully developed curved-tube flow.

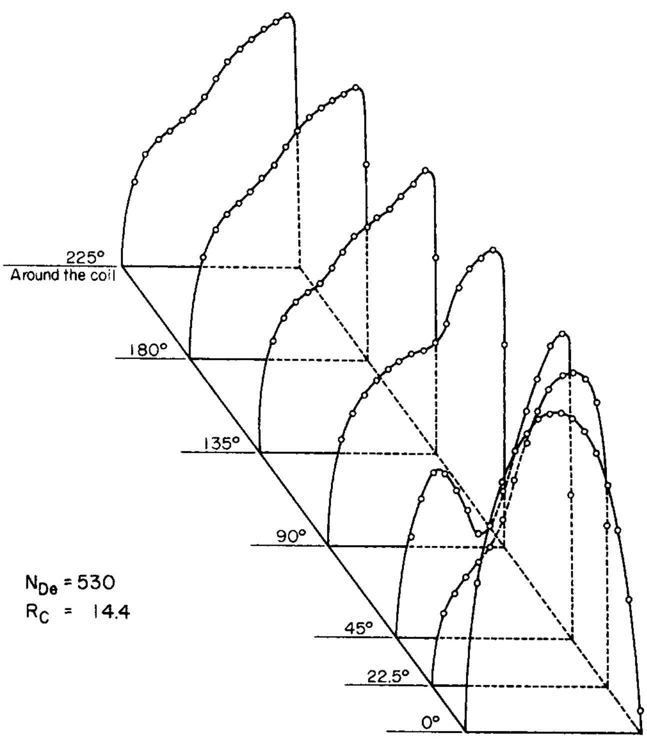

Fig. 1. Development of the axial velocity profile on a diameter in the direction of the radius of curvature.

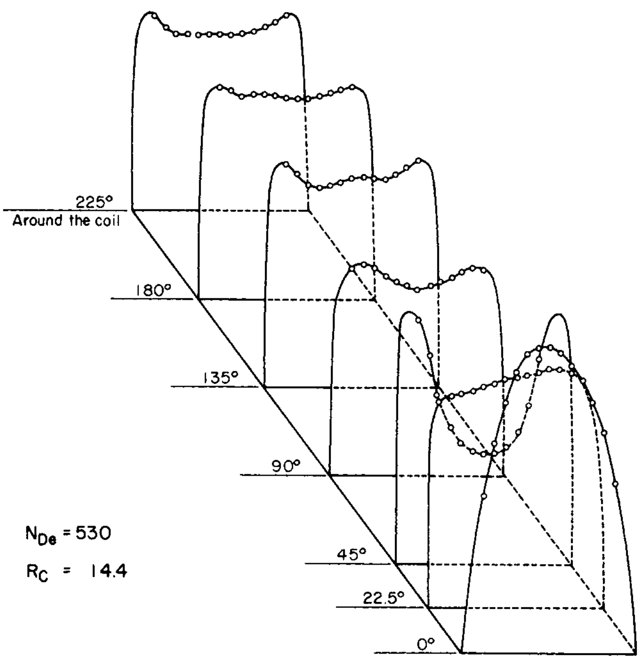

Fig. 2. Development of the axial velocity profile on a diameter in a direction perpendicular to the radius of curvature.

At the next measuring port of 45° around the coil, the most dramatic change takes place. Because the maximum velocity was near the outside wall at the station just before this one, the secondary velocity is responsible for bringing fluid of larger velocity from the outside of the coil to the inside. This results in the second peak occurring towards the inside of the coil. After this major change, the profile becomes settled as it successively approaches the typical skewed profile characteristic of fully-developed toroidal flow.

Figure 2 presents axial velocity profile data measured across the pipe diameter in a direction perpendicular to the direction of the coil radius, and thus to the profiles of Figure 1. These profiles also show a dramatic development. Again the axial velocity profile at the entrance to the coil is essentially parabolic. The profile at 22.5° around the coil is almost flat in the center. At 45°, the velocity in the core is severely depressed, in sharp contrast with the profile at 0°. By 90°, the relatively flat fully developed profile is emerging.

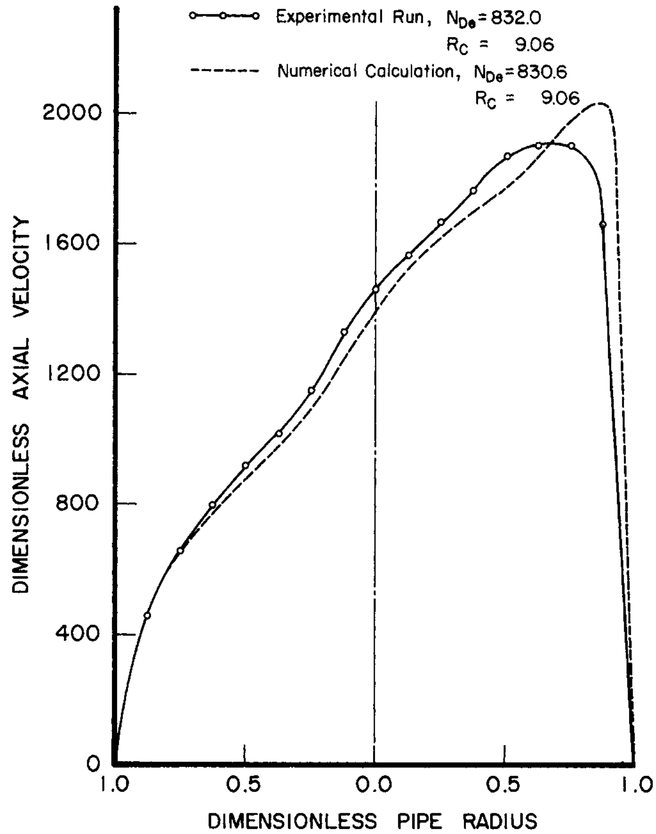

Fig. 3. Comparison of experimental and calculated fully developed axial velocity profile on a diameter in the direction of the radius of curvature.

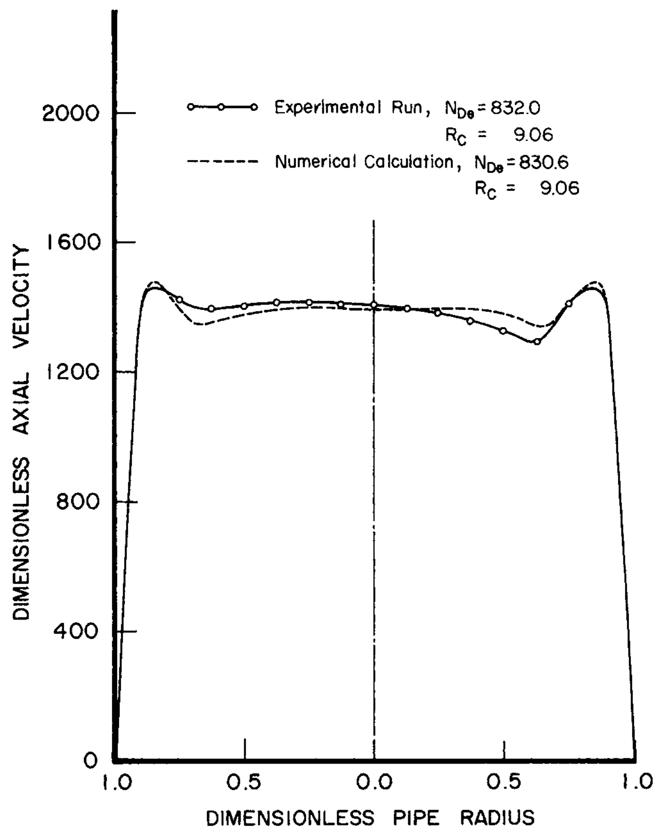

Fig. 4. Comparison of experimental and calculated fully developed axial velocity profile on a diameter in a direction perpendicular to the radius of curvature.

Comparisons of experimental fully developed axial velocity profiles were made with profiles calculated numerically by the method described by Austin and Seader (1973). Results are shown in Figures 3 and 4 for a Dean number of approximately 830 and a curvature ratio of 9.06. The relatively small disagreement is believed due to a combination of factors, including experimental and numerical errors, a pipe-coil cross section that was slightly flattened during bending to form the coil, and the coil pitch.

ENTRANCE-LENGTH CORRELATION

A useful result that was obtained from the experimental data on the development of the axial velocity profile was the length of coiled pipe (or number of degrees around the coil) required to achieve fully developed curved-pipe viscous flow. The flow was assumed to be fully developed when the basic shape of the axial velocity profile taken across the tube diameter from the inside to the outside of the coil remained unchanged. Once this rather skewed profile became constant with increasing length around the coil, it was reasoned that the secondary velocity, which is responsible for the change in shape of the primary axial velocity profile, should also have ceased to change.

Several methods were used to ascertain the location of fully developed flow. The one which gave the most consistent results was to plot the ratio of the axial velocity (in terms of the anemometic voltage) at a grid point near the wall where the peak velocity was ultimately achieved to the axial velocity at a grid point near the center of the tube against the number of degrees around the coil. In this manner, smooth curves were obtained which asymptotically approached a constant velocity ratio. The nature of the curves as shown by Austin (1971) allowed the entrance length to be determined to within 10° for each set of experimental data. The determined entry lengths varied from 90° for the smallest Dean number and largest curvature ratio to 245° for the largest Dean number and the smallest curvature ratio. Data from 12 runs were correlated with an average deviation of 1.7% and a maximum deviation of 5% by the empirical equation:

$$\gamma_D = 49\ (N_{De}/R_c)^{1/3}$$

where γ_D is the number of degrees around the coil to achieve fully developed curved-pipe flow, N_{De} is the Dean number, and R_c is the curvature ratio. This equation is also in good agreement with the experimental data of Keulegan and Beij (1937) for very large curvature ratios of 1435 to 4900.

ACKNOWLEDGMENT

This research was supported primarily through a National Defense Education Act Fellowship in the Chemical Engineering Department at the University of Utah.

LITERATURE CITED

Austin, L. R., "The Development of Viscous Flow Within Helical Coils," Ph.D. thesis, University of Utah, Salt Lake City (1971).

———., and J. D. Seader, "Fully Developed Viscous Flow in Coiled Circular Pipes," *AIChE J.*, **19**, 85 (1973).

Kalb, C. E., "Heat Transfer in Curved Tubes," Ph.D. thesis, University of Utah, Salt Lake City (1973).

Keulegan, G. H., and K. H. Beij, "Pressure Losses for Fluid Flow in Curved Pipes," *J. Res. Nat. Bur. Stand.*, **18**, 89 (1937).

Manuscript received December 13, 1973; revision received January 21 and accepted January 22, 1974.